Mr. Michael T. Davis Calculus	Chapter 2 – Practice Test November 10, 2016

Name: Solutions

1. Evaluate each limit

a. $\lim_{x \to 2} 3 =$ 3

b. $\lim_{x \to 2} 3x =$ 6

c. $\cancel{\lim [3y] =}$ $\lim_{x \to 2} (3y) = 3y$ If $\lim_{x \to 2} (3x)$, then $\lim_{x \to 2} (3x) = 6$

d. $\lim_{x \to \frac{\pi}{3}} \cos x =$ $\cos \frac{\pi}{3} = \frac{1}{2}$

e. $\lim_{x \to \frac{\pi}{2}^+} \tan x =$ $-\infty$

f. $\lim_{x \to \frac{3\pi}{2}} \sin x =$ $\sin \frac{3\pi}{2} = -1$

g. $\lim_{x \to \pi} \cos x =$ $\cos \pi = -1$

h. $\lim_{x \to 5} \frac{x-5}{x+1} =$ $\frac{5-5}{5+1} = \frac{0}{6} = 0$

i. $\lim_{x \to 5^+} \frac{x+1}{x-5} =$ ∞

j. $\lim_{x \to 5^-} \frac{x+1}{x-5} =$ $-\infty$

k. $\lim_{x \to 5} \frac{x+1}{x-5} =$ undefined

l. $\lim_{x \to +\infty} \frac{x+1}{x-5} = 1$

m. $\lim_{x \to -3} \frac{x^2-9}{x+3} = \lim_{x \to -3} \frac{(x-3)(x+3)}{x+3} = \lim_{x \to -3} (x-3) = -6$

n. $\lim_{t \to 0} \frac{\sin t}{t} = 1$

o. $\lim_{t \to \infty} \frac{\sin t}{t} = 0$

p. $\lim_{x \to 5} \frac{\sqrt{x}-\sqrt{5}}{x-5} = \lim_{x \to 5} \frac{\sqrt{x}-\sqrt{5}}{(\sqrt{x}-\sqrt{5})(\sqrt{x}+\sqrt{5})} = \lim_{x \to 5} \frac{1}{\sqrt{x}+\sqrt{5}} = \frac{1}{\sqrt{5}+\sqrt{5}} = \frac{1}{2\sqrt{5}}$

q. $\lim_{t \to 0} \frac{\sin(2t)}{t} = \lim_{t \to 0} \frac{2\sin t \cos t}{t} = 2\lim_{t \to 0} \frac{\sin t}{t} \cdot \lim_{t \to 0} \cos t = 2(1)(1) = 2$

r. Given $f(t) = \begin{cases} 2t+4 \text{ if } t \leq -1 \\ -3t-2 \text{ if } t > -1 \end{cases}$, $\lim_{t \to -1^+} f(t) = -3(-1)-2 = 3-2 = 1$

s. Given $f(t) = \begin{cases} 2t+4 \text{ if } t \leq -1 \\ -3t-2 \text{ if } t > -1 \end{cases}$, $\lim_{t \to -1} f(t) =$ undefined

t. Given $g(x) = \begin{cases} 5-x \text{ if } x < 2 \\ \frac{1}{2}x+2 \text{ if } x \geq 2 \end{cases}$, $\lim_{x \to 2} g(x) = 3$

2. Write an extended function for $f(t)=\dfrac{\sin t}{t}$ that is continuous at $t=0$.

$$g(t)=\begin{cases}\dfrac{\sin t}{t} & \text{if } t\neq 0\\ 1 & \text{if } t=0\end{cases}$$

3. Write an extended function for $f(t)=\dfrac{t^3-27}{t-3}$ that is continuous at $t=3$.

$$\frac{(t-3)(t^2+3t+9)}{t-3} \overset{\text{almost}}{=} t^2+3t+9$$

$$g(t)=\begin{cases}\dfrac{t^3-27}{t-3} & \text{if } t\neq 3\\ 27 & \text{if } t=3\end{cases}$$

4. Write an extended function for $g(y)=\dfrac{y^2-3y+2}{y^2-4}$ that is continuous at $y=2$.

$$\frac{(y-1)(y-2)}{(y+2)(y-2)} \overset{\text{almost}}{=} \frac{y-1}{y+2}$$

$$g(y)=\begin{cases}\dfrac{y^2-3y+2}{y^2-4} & \text{if } y\neq 2\\ \dfrac{1}{4} & \text{if } y=2\end{cases}$$

5. Find an end behavior model for each rational function, evaluate $\lim\limits_{x\to\infty} f(x)$ and determine equations for any horizontal or vertical asymptotes.

a. $f(x)=\dfrac{6x^3-1}{3x^3+4x}$ EBM $g(x)=2$ $\lim\limits_{x\to\infty} f(x)=2$ H.A. $y=2$ No V.A.

b. $f(x)=\dfrac{x^4-3}{2x^3-8x}$ EBM $g(x)=\frac{1}{2}x$ $\lim\limits_{x\to\infty} f(x)=\infty$ No H.A. V.A. $x=0$, $x=2$, $x=-2$

$2x(x^2-4)=2x(x-2)(x+2)$

c. $f(x)=\dfrac{4x^2+x}{x^3+8}$ EBM $g(x)=\frac{4}{x}$ $\lim\limits_{x\to\infty} f(x)=0$ H.A $y=0$ V.A. $x=-2$

$(x+2)(x^2-2x+4)$

6. A small rock is dropped from the top of a tall cliff. What is the average speed in feet per second of the falling rock from time $t=2$ seconds to $t=3$ seconds, i.e., during the third second of its fall? The equation $y=16t^2$ defines the number of feet the object falls during the first t seconds.

Average speed $=\dfrac{16(3)^2-16(2)^2}{3-2}=\dfrac{144-64}{1}=$

7. Find all values of k for which the function $f(x)=\begin{cases}x+4 & \text{if } x\le 1\\ x^2-2x+k & \text{if } x>1\end{cases}$ is continuous.

$(1)+4=(1)^2-2(1)+K$

$5=-1+K$

$6=K$

8. Find the average rate of change of $p(x)=3x^2$ over the interval.

a. Over the interval $[0,1]$ $\dfrac{3(1)^2-3(0)^2}{1-0}=3$

b. Over the interval $[1,3]$ $\dfrac{3(3)^2-3(1)^2}{3-1}=\dfrac{27-3}{2}=\dfrac{24}{2}=12$

$$\lim_{h\to 0} \frac{12h+3h^2-h}{h} = \lim_{h\to 0} \frac{11h+3h^2}{h} = \lim_{h\to 0}(11+3h) = 11$$

9. Find a right end behavior model and a left end behavior model for $f(x) = \sin x + e^x$.

right end $g(x) = e^x$ left end $K(x) = \sin x$

10. Find the instantaneous rate of change of $p(x) = 3x^2 - x$ at $x = 2$.

$$\lim_{h\to 0} \frac{f(2+h)-f(2)}{2+h-2} = \lim_{h\to 0} \frac{3(2+h)^2-(2+h)-(3(2)^2-2)}{h} = \lim \frac{12+12h+3h^2-2-h-10}{h}$$

11. Find the instantaneous rate of change of the surface area $S = 6x^2$ of a cube with respect to the edge length x at $x = a$.

$$\lim_{h\to 0} \frac{6(a+h)^2-6(a)^2}{a+h-a} = \lim_{h\to 0} \frac{6a^2+12ah+6h^2-6a^2}{h} = \lim_{h\to 0} \frac{12ah+6h^2}{h} = \lim_{h\to 0}(12a+6h)$$
$= 12a$

12. Given the function $f(x) = 3x^2 - x$. Determine:

$$\lim_{h\to 0} \frac{3(a+h)^2-(a+h)-(3a^2-a)}{a+h-a}$$

a. The slope of the curve at $x=1$

$$\lim_{h\to 0} \frac{3a^2+6ah+3h^2-a-h-3a^2+a}{h} = \lim_{h\to 0} \frac{6ah+3h^2-h}{h} = \lim_{h\to 0}(6a+3h-1)$$
$= 6a-1 = 6x-1$

At $x=1$ the slope is $6(1)-1 = 5$

b. An equation of the tangent to the curve at $x=1$

$f(1) = 3(1)^2-1 = 2$ $y-2 = 5(x-1)$

$m = 5$

c. An equation of the normal to the curve at $x=1$

$f(1) = 2$ $y-2 = -\frac{1}{5}(x-1)$

$m = -\frac{1}{5}$

13. Determine all types of discontinuity for each function.

a. $f(x) = \frac{x^2-81}{x-9} = \frac{(x-9)(x+9)}{x-9}$

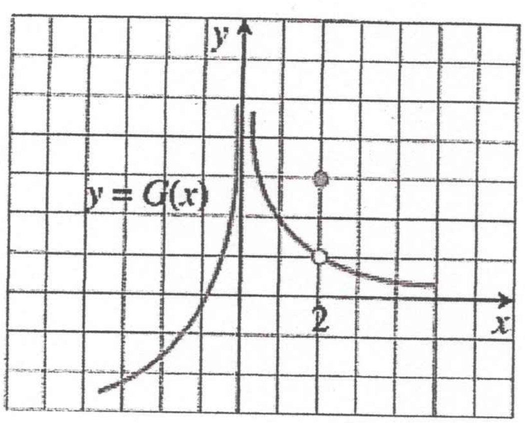
f(x) has a removable discontinuity at $x=9$

b. $g(x) = \frac{x-9}{x^2-81} = \frac{x-9}{(x-9)(x+9)}$

f(x) has a removable discontinuity at $x=9$ and an asymptotic continuity at $x=-9$

14. Given the graph of $y=G(x)$, determine each of the following:

a. $G(2) =$ 3



b. $\lim_{x\to 2} G(x) =$ 1

15. State which functions are by definition continuous functions and which are not continuous functions.

a. $g(x) = x^{\frac{1}{3}}$ continuous over its domain

b. $k(x) = \sqrt{x}$ continuous over its domain

c. $m(x) = \frac{1}{x}$ continuous over its domain

d. $p(x) = \frac{|x|}{x}$ continuous over its domain

e. $a(x) = \tan x$ continuous over its domain

16. Give a complete and succinct definition of a continuous function.

A function is continuous at a domain element $x = a$ if $\lim_{x \to a} f(x) = f(a)$

17. Sketch the graph of a function satisfying all of the given conditions:

(2,5) (4,4)

$\lim_{x \to -3^-} g(x) = -\infty$, $\lim_{x \to -3^+} g(x) = \infty$, $g(-1) = 0$, $g(0) = 1$,

$\lim_{x \to 2^-} g(x) = 3$, $g(2) = 5$, $\lim_{x \to 2^+} g(x) = 3$, $\lim_{x \to 4^-} g(x) = 2$, $\lim_{x \to 4^+} g(x) = 4$, $g(4) = 4$